Storm Surge Data Assimilation

Aijun Zhang, Eugene Wei, and Jye Chen
NOAA/National Ocean Service, N/CS13
1315 East West Highway
Silver Spring, MD 20910

Phone: (301) 713-2809 x 102
E-mail: eugene.wei@noaa.gov

Abstract

An optimal adjoint variational data assimilation technique has been developed to assimilate storm surges sampled along the U.S. East Coast into the two-dimensional Princeton Ocean Model (POM). This scheme uses the wind drag coefficient as the control variable since the storm surge at the coast are assumed to be predominately determined by the wind set-up on the shelf. In the optimal data assimilation procedure, the water level misfit is defined as the cost function and it's gradient can be determined by the adjoint model. In turn, the limited memory Broyden-Fletcher-Goldfarb-Shanno (BFGS) (Liu and Nocedal, 1989) quasi-Newton optimization method is implemented to search for the optimal wind drag coefficient by minimizing the cost function for the large scale optimization.

The data assimilation system was tested by identical experiments in which the pseudo-observations are generated by the numerical model with predefined wind drag coefficients. The results show that the wind drag coefficients can be recovered from observed water levels very accurately by using this adjoint optimal data assimilation system. The model is then applied to assimilate storm surges caused by hurricane Floyd, September 1999, along the U.S. East Coast. The model is forced by wind/pressure fields of the meso-scale NCEP's Eta Data Assimilation System (EDAS) 32km wind. Surge heights at 18 NOS's water level stations are assimilated into the model to produce the optimal coastal ocean water level nowcasts which serves as the initial conditions for the forecast model which, in turn, can provide the water level boundary conditions to a bay or harbor water level forecast model.

Introduction

Storm surges produced by strong storms in the U.S. East Coast cause property and structural damage and human life lost. A meteorologically forced (strong wind stress and atmospheric pressure depression) long wave motion, the extreme sustained storm surge increases the water surface elevations about the astronomical tide, causing innundation in low-lying coastal areas. The accurate and timely prediction of storm surges become critical in evacuation and rescue planning for saving human life.

The storm surge modeling has been developed and advanced for more than 30 years (Bode, et.al., 1997). Innovative developments in the storm surge prediction, for example, the coupled surge-wave modeling and data assimilation, have been included in several operational storm surge systems implemented in western Europe (Gerritsen, et. al., 1995). Traditionally, the storm surge modeling has been treated as a initial boundary value problem with the forward integration of a set of partial differential equations, usually the equations of continuity and momentum. Conventional model calibration and verification procedures are conducted by adjusting model parameters including the bathymetry, bottom friction and wind drag coefficients, and the diffusivity. The model deficiency, however, always exists due to many factors such as grid resolution and boundary forcing accuracy. The storm surge forecasts rely upon the numerical weather predictions (NWP) for model surface forcing. Therefore, the accuracy of NWP becomes critical to the storm surge modeling success.

The inverse methods are then introduced to improve the model performance by dealing with errors between model results and reliable measurements. The adjoint method and Kalman filtering are the most common approaches of the inverse method. In this study, the adjoint approach is adopted to adjust the wind drag coefficients by minimizing the differences between the modeled and observed water levels.

The development of the adjoint model for the two-dimensional barotropic Princeton Ocean Model (POM, Blumberg and Mellor, 1987) is described. The models are then applied to simulate storm surges, produced by Hurricane Floyd, during September 13 to 19, 1999 along the U.S. East Coast.

Hydrodynamic Model

The governing equations of two-dimensional POM are given as follows:

$$\frac{\partial \eta}{\partial t}+\frac{\partial UD}{\partial x}+\frac{\partial VD}{\partial y}=0 \qquad (1)$$

$$\frac{\partial UD}{\partial t}+\frac{\partial U^2D}{\partial x}+\frac{\partial UVD}{\partial y}-F_x-fVD+gD\frac{\partial \eta}{\partial x}=\frac{1}{\rho}\left(\tau_{sx}-\tau_{bx}\right) \quad (2)$$

$$\frac{\partial VD}{\partial t}+\frac{\partial UVD}{\partial x}+\frac{\partial V^2D}{\partial y}-F_y+fUD+gD\frac{\partial \eta}{\partial y}=\frac{1}{\rho}\left(\tau_{sy}-\tau_{by}\right) \quad (3)$$

where H, τ_s and τ_b are water depth at rest, wind stress and bottom friction, and $D=H+\eta$ total water depth, g the acceleration due to gravity, f the Coriolis parameter, ρ the water density. And the horizontal viscosity and diffusion terms F_x and F_y are defined as

$$F_x=\frac{\partial}{\partial x}\left[H2A_M\frac{\partial U}{\partial x}\right]+\frac{\partial}{\partial y}\left[HA_M\left(\frac{\partial U}{\partial y}+\frac{\partial V}{\partial x}\right)\right] \quad (4)$$

$$F_y=\frac{\partial}{\partial y}\left[H2A_M\frac{\partial V}{\partial y}\right]+\frac{\partial}{\partial x}\left[HA_M\left(\frac{\partial U}{\partial y}+\frac{\partial V}{\partial x}\right)\right] \quad (5)$$

where A_M, the vertically integrated horizontal eddy viscosity, is defined by the Smagorinsky formula

$$A_M=C\Delta x\Delta y\frac{1}{2}\left|\nabla V+\nabla V^T\right| \quad (6)$$

where C, a non-dimensional parameter, is set to be 0.2 in this study; Δx and Δy are the grid spacings in the x and y directions for each grid cell.

To simplify the adjoint model development, the fully non-linear two dimensional POM is linearized by: (1) neglecting the variation of surface elevation η relative to water depth (D=H); (2) neglecting the horizontal advection and diffusion terms (F_x and F_y); (3) linearizing the bottom friction terms with a constant bottom friction coefficient, $C_b=1.0\times10^{-3}$. Thus, the linearized 2-D POM governing equations are as follows,

$$\frac{\partial h}{\partial t}+\frac{\partial HU}{\partial x}+\frac{\partial HV}{\partial y}=0 \quad (7)$$

$$\frac{\partial HU}{\partial t}-fHV+gH\frac{\partial h}{\partial x}+C_d\left|U_w\right|\cdot U_w-C_bU=0 \quad (8)$$

$$\frac{\partial HV}{\partial t}+fHU+gH\frac{\partial h}{\partial y}+C_d\left|U_w\right|\cdot V_w-C_bV=0 \quad (9)$$

Adjoint Model

For the linearized 2-D POM with well-posed initial and open boundary conditions , the procedure of deriving adjoint equations is presented. The basic procedure in the variational adjoint method consists of minimizing a cost function that represents the misfits between observed data and model output. This minimization is performed subject to the strong constraints of satisfying the governing equations. The constraint minimization involves Lagrange multipliers and leads to additional equations (known as adjoint equations) from which Lagrange multipliers are determined. The model state variables and Lagrange multipliers are used to compute the cost function and its gradient from which the cost function is minimized to obtain the optimal control variables.

In this variational problem, the cost function defined as,

$$J=\frac{1}{2}\iiint_{xyt} W(h-h_o)^2\,dx\,dy\,dt \quad (10)$$

where h_o and h are observed and simulated elevations and W is the weighting factor. The variational problem is to minimize cost function J subject to equations (7)-(9). Introducing Lagrange multipliers λ_h, λ_u, λ_v for the constraint governing equations (7), (8), (9) (Lawson et al, 1995), the first variation of the cost function J can be written as

$$\delta J=\iiint_{xyt}\Big[\ (h-h_o)\quad+\lambda_h\delta(\frac{\partial h}{\partial t}+\frac{\partial HU}{\partial x}+\frac{\partial HV}{\partial y})$$
$$+\lambda_u\delta(\frac{\partial HU}{\partial t}-fHV+gH\frac{\partial h}{\partial x}+C_d\left|U_w\right|\cdot U_w-C_bU) \quad (11)$$
$$+\lambda_v\delta(\frac{\partial HV}{\partial t}+fHU+gH\frac{\partial h}{\partial y}+C_d\left|U_w\right|\cdot V_w-C_bV)\ \Big]dx\,dy\,dt$$

The corresponding adjoint variables are defined as follows: λ_h is the adjoint variable of h, λ_u is the adjoint variable of U, and λ_v is the adjoint variable of V, respectively.

Considering the specific case defined above, using wind drag coefficients as the only control variables, the adjoint equations are expressed as follows,

$$H\frac{\partial \lambda_u}{\partial t}-fH\lambda_v+H\frac{\partial \lambda_h}{\partial x}+C_b\lambda_u=0 \quad (12)$$

$$H\frac{\partial \lambda_v}{\partial t}+fH\lambda_u+H\frac{\partial \lambda_h}{\partial y}+C_b\lambda_v=0 \quad (13)$$

$$\frac{\partial \lambda_h}{\partial t}+g(\frac{\partial H\lambda_u}{\partial x}+\frac{\partial H\lambda_v}{\partial y})=h-h_o \quad (14)$$

and the function increment becomes

$$\delta J=\iiint\left(\lambda_u\left|U_w\right|U_w+\lambda_v\left|U_w\right|V_w\right)\delta C_d\,dxdydt \quad (15)$$

The above process showed that cost function minimization has resulted in new equations (12)-(14) which are called adjoint equations (Zhang, et.al., 2000). And the adjoint variables are calculated by integrating the adjoint equations. The gradient of the cost function with respect to the control variable, wind drag coefficient C_d, can be computed by (16). It is shown from (16) that, if C_d varies spatially and temporally, the gradients of cost function with respect to C_d can be computed by integrating adjoint model once regardless of the number of control variables.

$$G=\iiint_{xyt}\left(\lambda_u\left|U_w\right|U_w+\lambda_v\left|U_w\right|V_w\right)dxdydt \quad (16)$$

An iterative scheme for 24 hours data assimilation is given in Fig.1.

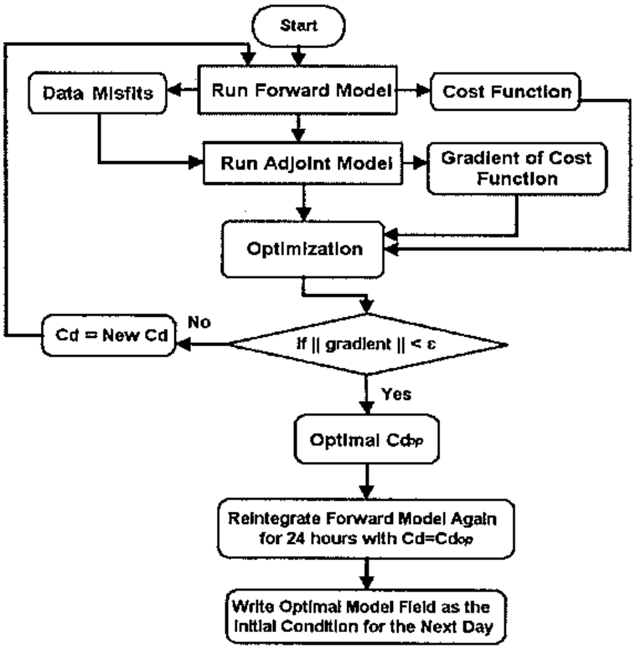

Fig. 1. Flow Chart for a 24-Hour Water Level Data Assimilation

Simulations

The orthogonal curvilinear model grid (Fig.2), dimensioned 120 by 85, covers the coastal waters in the U.S. East Coast with a grid resolution of 5 to 32 km. The model is forced with US NWS/NCEP's Eta Data Assimilation System (EDAS) 32km wind analyses. Wind vectors for Floyd at 00Z, September 16 interpolated to the model grid are shown in Figs. 3.

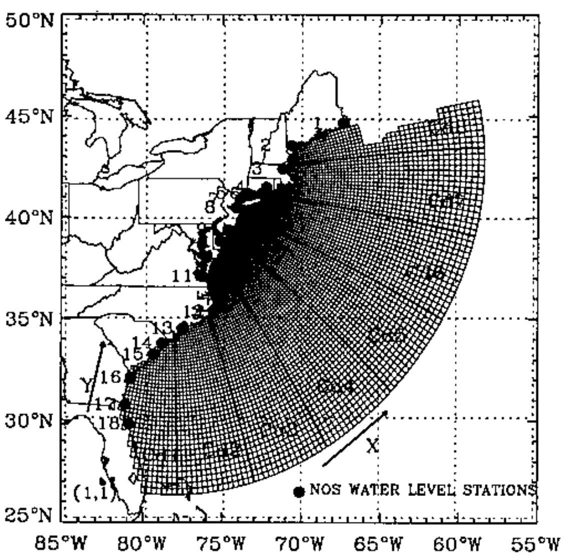

Fig. 2. Model grid and NOS water level station location.

The forward storm surge simulation started from rest at September 5, 1999 (Julian Day 248) for 7 days without data assimilation to produce a restart file describing the entire model dynamics as the initial condition for the data assimilation. The adjoint model started to assimilate water levels on September 12 (Julian Day 255) an continued to September 19 (Julian Day 262).

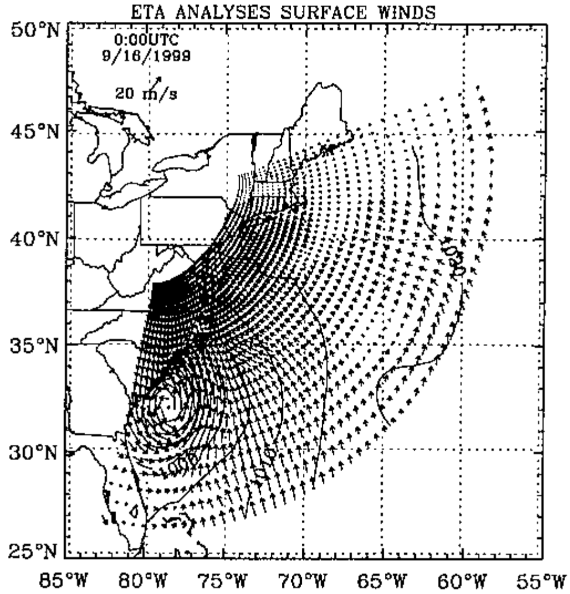

Fig. 3. EDAS surface wind vectors.

<u>Twin Experiment</u>

In order to check the performance of adjoint optimal data assimilation technique, identical twin experiments are usually considered. In the twin experiment, the pseudo-observation data are generated by the numerical model. This is the best situation for data assimilation since the pseudo-observational data contain the same dynamics as the numerical model without other errors. In this study, the pseudo-subtidal water levels are generated by integrating the model with 8 predefined time-dependent sub-regional wind drag coefficients (definition of C_d is shown in Fig. 2). Hourly model data are sub-sampled at 18 selected locations. These data are treated as pseudo-observations in the subsequent data assimilation with an arbitrary C_d value as a first guess. Fig. 5 shows the optimal C_d, obtained at the end of each 24-hours data assimilation window, converges to the predefined values relatively fast and the differences are in the order of 10^{-6}. This test verifies that the adjoint model works properly.

<u>Data Assimilation</u>

Hourly observations during Floyd at 18 NOAA/NOS water level stations along the U.S. East Coast are processed to remove the astronomical signals. The de-tided signals (Fig.6 solid line) are then assimilated into the adjoint model. The model spun off for 7 days (Julian

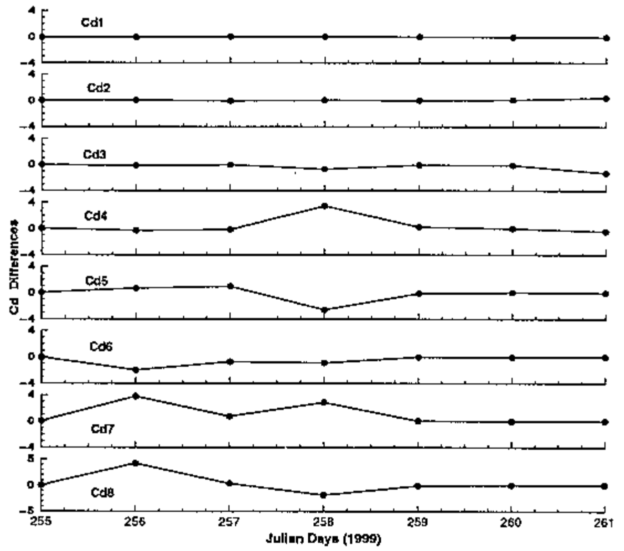

Fig. 4. Difference between optimal and predefined Cd (x10^6) in the identical twin experiment.

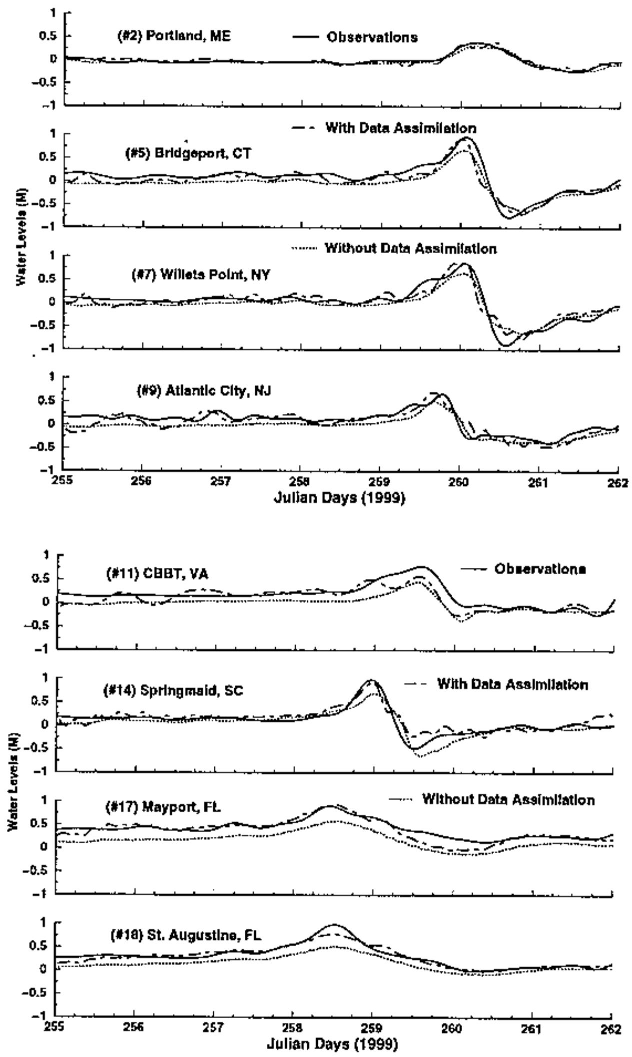

Fig. 5. Water levels with (dot-dashed line) and without (dotted lines) data assimilation compared with observations (solid lines) at 8 locations.

Day 248 to 255) from rest before assimilating the water levels. Fig.6 shows the simulated water levels with (dot-dashed lines and without (dotted lines) the data assimilation and the observations (solid lines) at 8 representative stations from north to south. The surge height reached about 1 m at South Carolina coast where Floyd landed. Water levels are amplified at Bridgeport, CT and Willets Pt., NY when surge wave entered Long Island Sound. In most stations, the simulated water levels with the data assimilation are much closer to the observations than the no data assimilation simulated water levels. However, the difference is insignificant at northeast stations such as Portland, Maine where the storm effect is minimal. The peak surge root-mean-square errors for all 18 stations are reduced from 30 cm without data assimilation to 22 cm with data assimilation.

An experiment has been conducted to examine the data assimilation scheme performance in simulating surge heights at locations where no observational station exists. In this experiment, the water level observations at Sandy Hook, NJ (#8) and Ft. Pulaski, GA (#16) are excluded from the data assimilation simulation. Fig.7 shows the simulated water level time history at these two stations from this experiment, observations, and with (including these stations) and without data assimilation. The simulated water level differences between the data assimilation including and excluding these two stations are insignificant. The simulated water levels at stations on both sides of these two stations show no difference for both cases. This experiment indicates that water levels at locations without measurements can be filled by assimilating the observations at nearby stations if they are subject to similar dynamic effects.

Conclusions

The variational adjoint model, using the wind drag coefficient as the control variable, has been developed base on the two-dimensional linearized POM. Observed water levels are used to adjust 8 wind drag coefficients, representing each model sub-regions, by minimizing the model simulated water level misfits. The adjoint model has been validated with a twin experiment and tested on a model grid covering coastal waters in US East Coast for simulating storm surges. The surge heights simulated by the adjoint model at locations without measurements are more accurate than that without data assimilation.

The adjoint model for water level simulations in shallow water area can be improved by including the non-linear terms such as advection and bottom friction. However, such a model would require more memory storage and computation. The wind stress direction can also be optimized by breaking the wind drag coefficient into each components, i.e., C_{dx} and C_{dy}. Preliminary test shows more accurate simulated water levels with additional wind stress direction adjustment. The open boundary condition can also be included in the adjoint model as a control variable. The astronomical tide can be included in conjunction with the boundary condition.

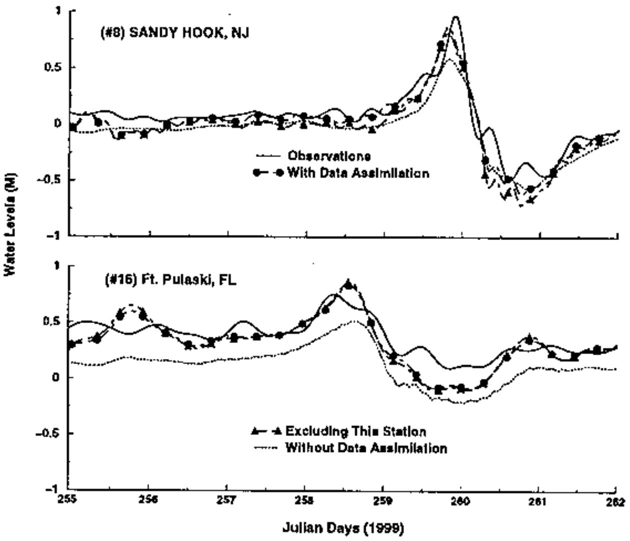

Fig. 6. Observed and simulated subtidal water levels at Sand Hook, NJ and Ft. Pulaski, GA with data assimilation (circle: including, Triangle: excluding, these two stations) and without data assimilation.

References

Blumberg, A. F. and G. L. Mellor, 1987. "A description of a three-dimensional coastal ocean circulation model." Three Dimensional Coastal Ocean Models. N. S. Heaper, Amer. Geophys. Union, 1-16.

Bode, L. and Hardy, T., 1997. "Progress and Recent Developments in Storm Surge Modeling", Journal of Hydraulic Engineering, Vol. 123, No. 4, April, 1997, ASCE.

Gerritsen, H., de Vries, H., and Philippart, M. (1995) "The Dutch continental shelf model." *Quantitative skill assessment for coastal ocean models*, D. R. Lynch and A. M. Davies, eds., Coastal and Estuarine Studies, Vol. 47, Am. Geophys. Union, Washington, D.C., 425-467.

Lawson, L. M, Y. h. Spitz, E. E. Hofmann, and R. B. Long. (1995). "A data assimilation technique applied to a predator-prey model. Bulletin of Mathematical Biology, 1995, 57, 593-617.

Liu, D. and J. Nocedal, 1989. "On the limited memory BFGS method for large scale optimization." Mathematical Programming, B 45, 503-528.

Zhang, A., E. Wei, and B. Parker, 2000. "Parameter estimation for Subtidal Water Levels Using an Adjoint Variational Optimal Method", In: Estuarine and Coastal Modeling, (in press) Malcolm L. Spaulding edits, ASCE.